REVIEW PROBLEMS

In Problems 1–30, solve the equation.

1. $\dfrac{dy}{dx} = \dfrac{e^{x+y}}{y-1}$

2. $\dfrac{dy}{dx} - 4y = 32x^2$

3. $(x^2 - 2y^{-3})dy + (2xy - 3x^2)dx = 0$

4. $\dfrac{dy}{dx} + \dfrac{3y}{x} = x^2 - 4x + 3$

5. $[\sin(xy) + xy\cos(xy)]dx + [1 + x^2\cos(xy)]dy = 0$

6. $2xy^3\,dx - (1 - x^2)dy = 0$

7. $t^3y^2\,dt + t^4y^{-6}\,dy = 0$

8. $\dfrac{dy}{dx} + \dfrac{2y}{x} = 2x^2y^2$

9. $(x^2 + y^2)dx + 3xy\,dy = 0$

10. $[1 + (1 + x^2 + 2xy + y^2)^{-1}]dx$
 $+ [y^{-1/2} + (1 + x^2 + 2xy + y^2)^{-1}]dy = 0$

11. $\dfrac{dx}{dt} = 1 + \cos^2(t - x)$

12. $(y^3 + 4e^xy)dx + (2e^x + 3y^2)dy = 0$

13. $\dfrac{dy}{dx} - \dfrac{y}{x} = x^2\sin 2x$

14. $\dfrac{dx}{dt} - \dfrac{x}{t-1} = t^2 + 2$

15. $\dfrac{dy}{dx} = 2 - \sqrt{2x - y + 3}$

16. $\dfrac{dy}{dx} + y\tan x + \sin x = 0$

17. $\dfrac{dy}{d\theta} + 2y = y^2$

18. $\dfrac{dy}{dx} = (2x + y - 1)^2$

19. $(x^2 - 3y^2)dx + 2xy\,dy = 0$

20. $\dfrac{dy}{d\theta} + \dfrac{y}{\theta} = -4\theta y^{-2}$

21. $(y - 2x - 1)dx + (x + y - 4)dy = 0$

22. $(2x - 2y - 8)dx + (x - 3y - 6)dy = 0$

23. $(y - x)dx + (x + y)dy = 0$

24. $\left(\sqrt{y/x} + \cos x\right)dx + \left(\sqrt{x/y} + \sin y\right)dy = 0$

25. $y(x - y - 2)dx + x(y - x + 4)dy = 0$

26. $\dfrac{dy}{dx} + xy = 0$

27. $(3x - y - 5)dx + (x - y + 1)dy = 0$

28. $\dfrac{dy}{dx} = \dfrac{x - y - 1}{x + y + 5}$

29. $(4xy^3 - 9y^2 + 4xy^2)dx$
 $+ (3x^2y^2 - 6xy + 2x^2y)dy = 0$

30. $\dfrac{dy}{dx} = (x + y + 1)^2 - (x + y - 1)^2$

In Problems 31–40, solve the initial value problem.

31. $(x^3 - y)\,dx + x\,dy = 0\,, \quad y(1) = 3$

32. $\dfrac{dy}{dx} = \left(\dfrac{x}{y} + \dfrac{y}{x}\right)\,, \quad y(1) = -4$

33. $(t + x + 3)dt + dx = 0\,, \quad x(0) = 1$

34. $\dfrac{dy}{dx} - \dfrac{2y}{x} = x^2\cos x\,, \quad y(\pi) = 2$

35. $(2y^2 + 4x^2)dx - xy\,dy = 0\,, \quad y(1) = -2$

36. $[2\cos(2x + y) - x^2]dx$
 $+ [\cos(2x + y) + e^y]dy = 0\,, \quad y(1) = 0$

37. $(2x - y)dx + (x + y - 3)dy = 0\,, \quad y(0) = 2$

38. $\sqrt{y}\,dx + (x^2 + 4)dy = 0\,, \quad y(0) = 4$

39. $\dfrac{dy}{dx} - \dfrac{2y}{x} = x^{-1}y^{-1}\,, \quad y(1) = 3$

40. $\dfrac{dy}{dx} - 4y = 2xy^2\,, \quad y(0) = -4$

41. Express the solution to the following initial value problem using a definite integral:

$$\frac{dy}{dt} = \frac{1}{1 + t^2} - y\,, \quad y(2) = 3\,.$$

Then use your expression and numerical integration to estimate $y(3)$ to four decimal places.

TECHNICAL WRITING EXERCISES

1. An instructor at Ivey U. asserted: "All you need to know about first-order differential equations is how to solve those that are exact." Give arguments that support and arguments that refute the instructor's claim.
2. What properties do solutions to linear equations have that are not shared by solutions to either separable or exact equations? Give some specific examples to support your conclusions.
3. Consider the differential equation

$$\frac{dy}{dx} = ay + be^{-x}, \qquad y(0) = c,$$

where a, b, and c are constants. Describe what happens to the asymptotic behavior as $x \to +\infty$ of the solution when the constants a, b, and c are varied. Illustrate with figures and/or graphs.